UNIT-4

SHAFTS

Introduction:-A shaft is a rotating machine element which transmit power from one place to another. Shaft are subjected to tensile, bending or torsional stresses or to a combination of these stresses.

A transmitting shaft is circular is cross section, which supports transmission elements like pulleys, gears and sprockets.

The design of transmission shaft consists of determining the corret shaft diameter based on 1). Strength, 2). Rigidity and Stiffness.

Axle: It is a non rotating shaft, which supports the rotating components of the machine. It does not transmit a useful torque. Axle is subjected to only bending.

S*pindle: It* is a short rotating shaft in case of drilling machine, lathe spindles.

Line shaft or transmission shaft: It is a comparatively long shaft which is driven by a motor. The line shaft transmits motion to various machines through counter shafts. The counter shaft is an intermediate shaft placed between the line shaft and various driven machines.

Stub axle: It is short axle capable of small angular motion about the pivots. Front wheels of rear wheel drive vehicles are supported on stub axles.

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.

2. It should have good machinability.

3. It should have low notch sensitivity factor.

4. It should have good heat treatment properties.

5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

Types of Shafts

The following two types of shafts are important from the subject point of view :

1. ***Transmission shafts*****.** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. ***Machine shafts*****.** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are : 25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps. The standard length of the shafts are 5 m, 6 m and 7 m.

Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(a) Shafts subjected to twisting moment or torque only,

(*b*) Shafts subjected to bending moment only,
(*c*) Shafts subjected to combined twisting and bending moments, and
(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{.....................}(i)$$

where T = Twisting moment (or torque) acting upon the shaft,
J = Polar moment of inertia of the shaft about the axis of rotation,
τ = Torsional shear stress, and
r = Distance from neutral axis to the outer most fibre
= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation **(*i*)** may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{................}(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).
We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.
Substituting these values in equation **(*i*)**, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{.........}(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o
Now the equation **(*iii*)** may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \text{...........}(iv)$$

From the equations **(*iii*)** or **(*iv*)**, the outside and inside diameter of a hollow shaft may be determined. It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :
We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\, R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and
R = Radius of the pulley.

Problems(1):- *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Given data : N = 200 r.p.m. ; P = 20 kW = 20 × 10³ W; τ = 42 MPa = 42 N/mm²

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

∴ $d^3 = 955 \times 10^3 / 8.25 = 115\,733$ or d = 48.7 say 50 mm

Problem(2):- *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Given data : P = 1 MW = 1 × 10⁶ W ; N = 240 r.p.m. ; $T_{max} = 1.2\ T_{mean}$; τ = 60 MPa = 60 N/mm²

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

∴ Maximum torque transmitted,

$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3$ N-mm

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

∴ $d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$

or $d = 159.4$ say 160 mm

Problem(3):- *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.*

Given data :P = 20 kW = 20 × 10³ W ; N = 200 r.p.m. ; τ_u = 360 MPa = 360 N/mm²

$F.S.$ = 8 ; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

∴ $d^3 = 955 \times 10^3 / 8.84 = 108\ 032$ or $d = 47.6$ say 50 mm

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\ (d_o)^3\ [1 - (0.5)^4] = 8.3\ (d_o)^3$$

∴ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\ 060$ or $d_o = 48.6$ say 50 mm

and $d_i = 0.5 d_o = 0.5 \times 50 = 25$ mm

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \text{..................}(\boldsymbol{i})$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (***i***), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \quad \text{........(where } k = d_i / d_o \text{)}$$

and $y = d_o / 2$

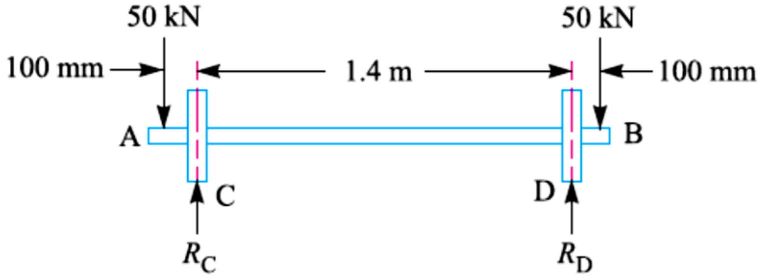
Again substituting these values in equation (***i***), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Problem(4):- *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box,acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Given data : W = 50 kN = 50 × 10^3 N ; L = 100 mm ; x = 1.4 m ; σ_b = 100 MPa = 100 N/mm^2



The axle with wheels is shown in Fig.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

> The maximum B.M. may be obtained as follows :
> $R_C = R_D = 50$ kN = 50×10^3 N
> B.M. at *A*, $M_A = 0$
> B.M. at *C*, $M_C = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm
> B.M. at *D*, $M_D = 50 \times 10^3 \times 1500 - 50 \times 10^3 \times 1400 = 5 \times 10^6$ N-mm
> B.M. at *B*, $M_B = 0$

M = *W.L* = 50 × 103 × 100 = 5 × 106 N-mm

Let d = Diameter of the axle.

We know that the maximum bending moment (*M*),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

∴ $d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6$ or $d = 79.8$ say 80 mm

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild
steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad \text{...............}(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation ***(i)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...............}(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad \text{...............}(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

$$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad \text{....................}(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad \text{....................}(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations ***(ii)*** and ***(v)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Problem(5):- *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

Given data : M = 3000 N-m = 3 × 10^6 N-mm ; T = 10 000 N-m = 10 × 10^6 N-mm ;

σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3\times10^6)^2 + (10\times10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

∴ d^3 = 10.44 × 10^6 / 16.36 = 0.636 × 10^6 or d = 86 mm

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}(M + T_e)$$

$$= \tfrac{1}{2}(3\times10^6 + 10.44\times10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

∴ d^3 = 6.72 × 10^6 / 11.46 = 0.586 × 10^6 or d = 83.7 mm

Taking the larger of the two values, we have

d = 86 say 90 mm

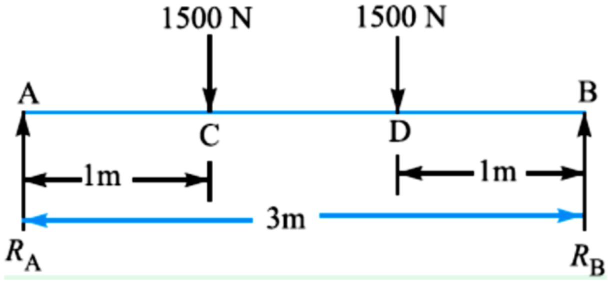
Problem(6):- *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Given data: P = 100 kW = 100 × 10^3 W ; N = 300 r.p.m. ; L = 3 m ; W = 1500 N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig.



The reaction at each support will be 1500 N,

i.e. $R_A = R_B = 1500$ N

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

∴ Maximum bending moment, $M = 1500 \times 1 = 1500$ N-m

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$

$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3$$ ....(Assuming $\tau = 60$ N/mm^2)

∴ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or $d = 66.8$ say 70 mm

Problem(7):- *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180^0 and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.*

Given data : AB = 1 m ; D_C = 600 mm or R_C = 300 mm = 0.3 m ; AC = 300 mm = 0.3 m ; T_1 = 2.25 kN = 2250 N ; D_D = 400 mm or R_D = 200 mm = 0.2 m ; BD = 200 mm = 0.2 m ; θ = 180° = π rad ; μ = 0.24 ; σ_b = 63 MPa = 63 N/mm^2 ; τ = 42 MPa = 42 N/mm^2

The space diagram of the shaft is shown in Fig (*a*).

Let T_1 = Tension in the tight side of the belt on pulley *C* = 2250 N ......(Given)

T_2 = Tension in the slack side of the belt on pulley *C*.

We know that

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

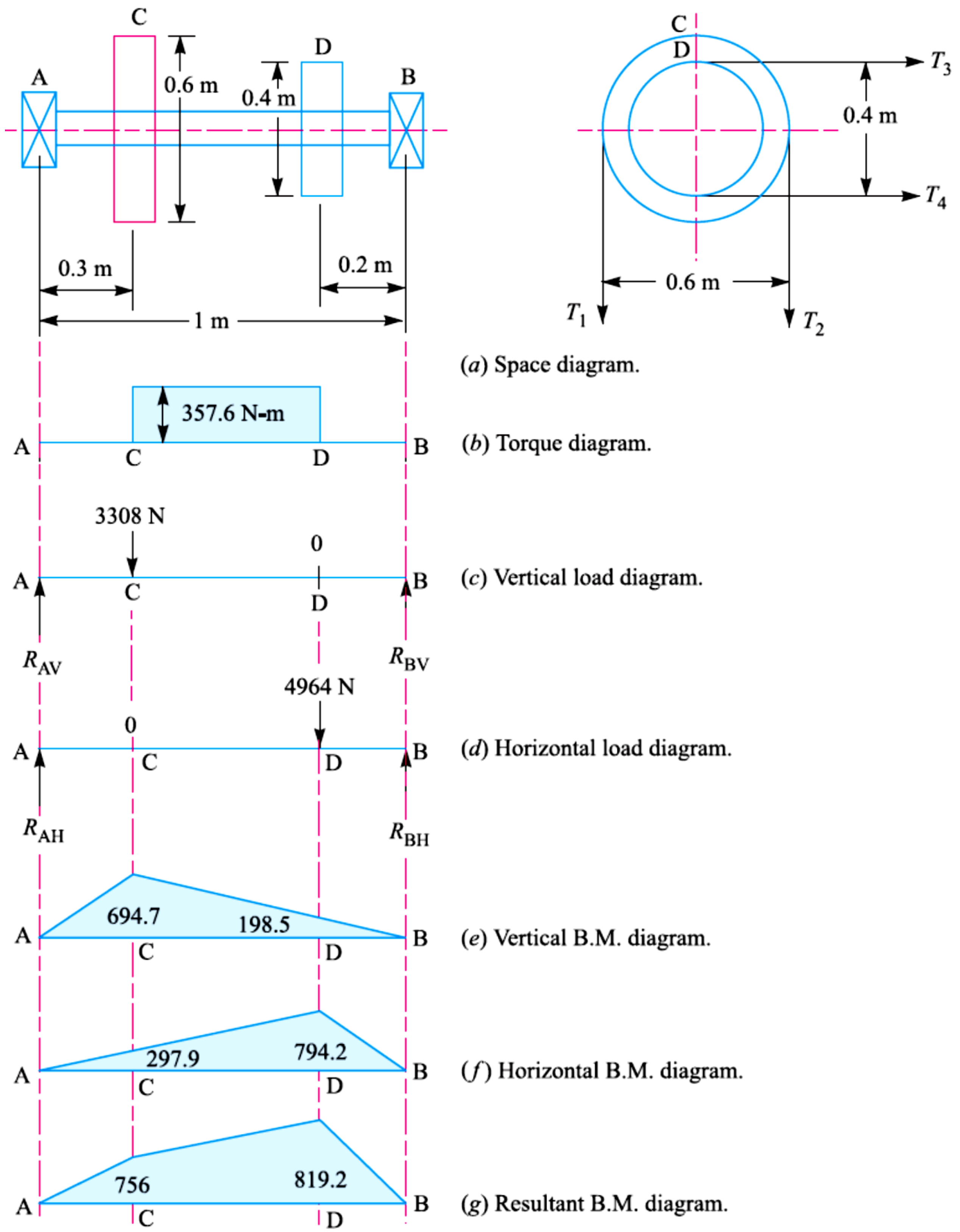
$$\frac{T_1}{T_2} = e^{0.24(\pi)} \quad = \quad \frac{T_1}{T_2} = 2.127$$

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

∴ Vertical load acting on the shaft at *C*,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at $D = 0$



(a) Space diagram.

(b) Torque diagram.

(c) Vertical load diagram.

(d) Horizontal load diagram.

(e) Vertical B.M. diagram.

(f) Horizontal B.M. diagram.

(g) Resultant B.M. diagram.

The vertical load diagram is shown in Fig (*c*).

We know that torque acting on the pulley *C*,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig (*b*).

Let T_3 = Tension in the tight side of the belt on pulley *D*, and

T_4 = Tension in the slack side of the belt on pulley *D*.

Since the torque on both the pulleys (*i.e*. *C* and *D*) is same, therefore

$$(T_3 - T_4)\ R_D = T = 357.6 \text{ N-m}$$

$$T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \text{................}(i)$$

We know that

$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \text{ or } T_3 = 2.127\ T_4 \qquad \text{................(ii)}$$

From equations (*i*) and (*ii*), we find that

T_3 = 3376 N, and T_4 = 1588 N

∴ Horizontal load acting on the shaft at *D*,

$W_D = T_3 + T_4 = 3376 + 1588 = 4964$ N

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig(*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$R_{AV} + R_{BV} = 3308$ N

Taking moments about *A*,

$R_{BV} \times 1 = 3308 \times 0.3$ or $R_{BV} = 992.4$ N

and $R_{AV} = 3308 - 992.4 = 2315.6$ N

We know that B.M. at *A* and *B*,

$M_{AV} = M_{BV} = 0$

B.M. at *C*, $M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7$ N-m

B.M. at *D*, $M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5$ N-m

The bending moment diagram for vertical loading in shown in Fig (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$R_{AH} + R_{BH} = 4964$ N

Taking moments about *A*,

$R_{BH} \times 1 = 4964 \times 0.8$ or $R_{BH} = 3971$ N

and $R_{AH} = 4964 - 3971 = 993$ N

We know that B.M. at *A* and *B*,

$M_{AH} = M_{BH} = 0$

B.M. at *C*, $M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9$ N-m

B.M. at *D*, $M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

The bending moment diagram for horizontal loading is shown in Fig(*f*).

Resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig (*g*).

We see that bending moment is maximum at *D*.

∴ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$

$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

∴ $d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3$ or $d = 47.6$ mm

Again we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

∴ $d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3$ or $d = 51.7$ mm

Taking larger of the two values, we have

d = 51.7 say 55 mm.

Problem(8):- *A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm module is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.*

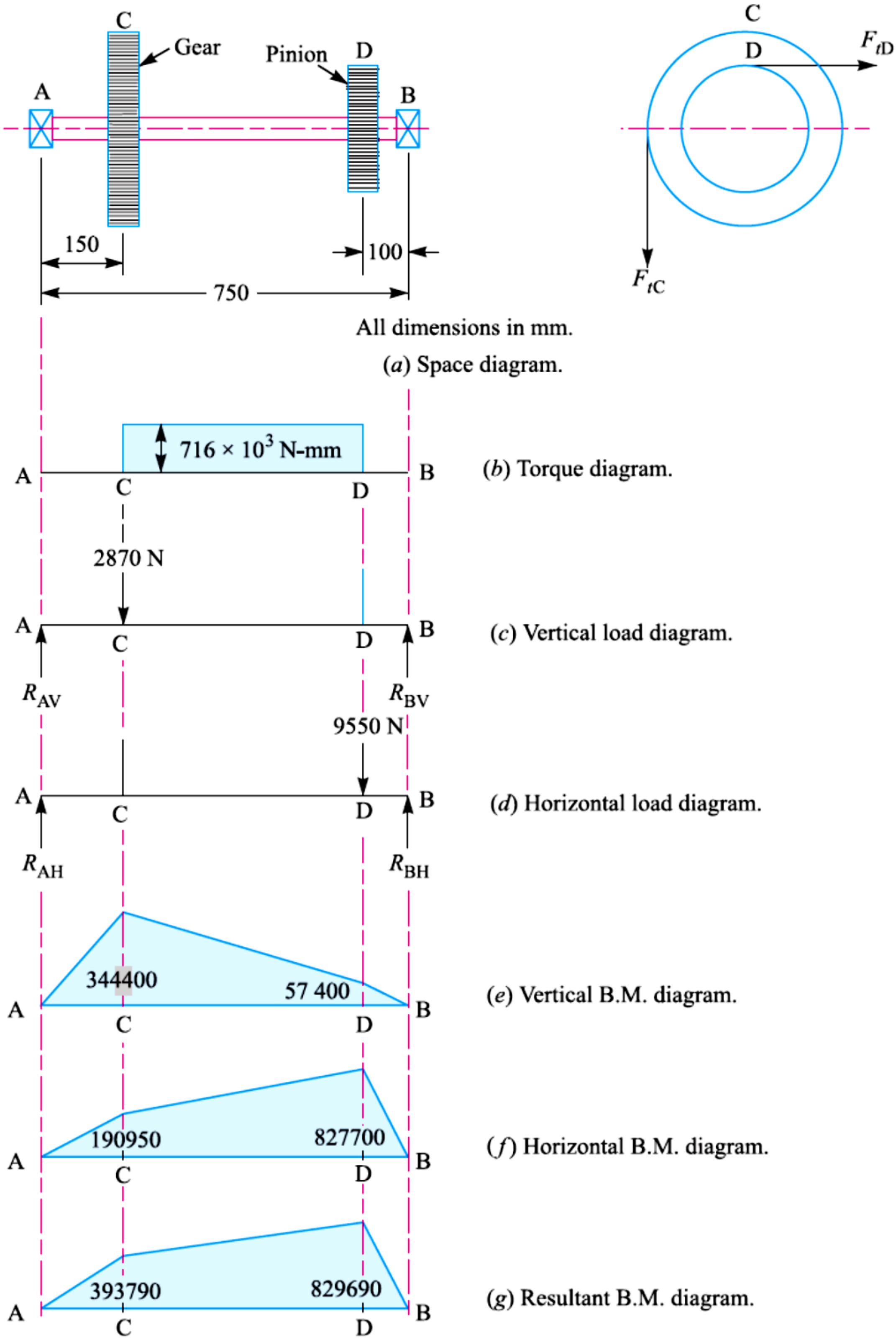
Given data : P = 15 kW = 15×10^3 W ; N = 200 r.p.m. ; AB = 750 mm ; $T_D = 30$; m_D = 5 mm ; BD = 100 mm ; $T_C = 100$; m_C = 5 mm ; AC = 150 mm ; τ = 54 MPa = 54 N/mm^2

The space diagram of the shaft is shown in Fig (*a*).

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

The torque diagram is shown in Fig (*b*).



(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagram.

We know that diameter of gear = No. of teeth on the gear × module

∴ Radius of gear *C*,

$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion *D*,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at *C* and *D* is same (*i.e.* 716 × 10^3 N-mm), therefore tangential force on the gear *C*, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion *D*, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig (*c*) and (*d*) respectively. Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 2870 \text{ N}$$

Taking moments about *A*, we get

$$R_{BV} \times 750 = 2870 \times 150$$

$$R_{BV} = 2870 \times 150 / 750 = 574 \text{ N}$$

and $R_{AV} = 2870 - 574 = 2296$ N

We know that

B.M. at *A* and *B*, $M_{AV} = M_{BV} = 0$

B.M. at *C*, $M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\,400$ N-mm

B.M. at *D*, $M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\,400$ N-mm

The B.M. diagram for vertical loading is shown in Fig (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about *A*, we get

$$R_{BH} \times 750 = 9550 (750 - 100) = 9550 \times 650$$

∴ $R_{BH} = 9550 \times 650 / 750 = 8277$ N

and $R_{AH} = 9550 - 8277 = 1273$ N

We know that

B.M. at A and B, $M_{AH} = M_{BH} = 0$

B.M. at C, $M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\,950$ N-mm

B.M. at D, $M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\,700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig (*f*).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\,400)^2 + (190\,950)^2}$$

$$= 393\,790 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\,400)^2 + (827\,700)^2}$$

$$= 829\,690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig (*g*). We see that the bending moment is maximum at D.

∴ Maximum bending moment, $M = M_D = 829\,690$ N-mm

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\,690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

∴ $d^3 = 1096 \times 10^3 / 10.6 = 103.4 \times 10^3$

or $d = 47$ say 50 mm.

Shafts Subjected to Fluctuating Loads:-

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M). Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

Problem(9):- *A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central* *load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?*

Given data : P = 20 kW = 20 × 10^3 W ; N = 200 r.p.m. ; W = 900 N ; L = 2.5 m ; τ = 42 MPa = 42 N/mm^2 ; σ_b = 56 MPa = 56 N/mm^2

Size of the shaft

Let d = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$

$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ d^3 = 1108 × 10^3 / 8.25 = 134.3 × 10^3 or d = 51.2 mm

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \tfrac{1}{2}(M + T_e)$$

$$= \tfrac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\ d^3$$

$\therefore$ d^3 = 835.25 × 10^3 / 5.5 = 152 × 10^3 or d = 53.4 mm

Taking the larger of the two values, we have

d = 53.4 say 55 mm

Size of the shaft when subjected to gradually applied load

Let d = Diameter of the shaft.

Assume for rotating shafts with gradually applied loads,

K_m = 1.5 and K_t = 1

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 562.5 \times 10^3)^2 + (1 \times 955 \times 10^3)^2} = 1274 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1274 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

∴ $d^3 = 1274 \times 10^3 / 8.25 = 154.6 \times 10^3$ or $d = 53.6$ mm

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\left[K_m \times M + T_e\right]$$

We also know that equivalent bending moment (M_e),

$$1059 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

∴ $d^3 = 1059 \times 10^3 / 5.5 = 192.5 \times 10^3 = 57.7$ mm

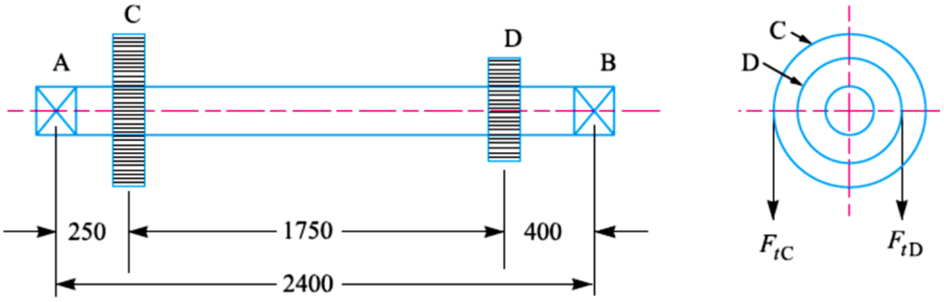
Taking the larger of the two values, we have

$d = 57.7$ say 60 mm

Problem(10):- *A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centre line of the bearings is 2400 mm. The shaft transmits 20 kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwards.*

Find the diameter of the shaft, if the working stress is 100 MPa in tension and 56 MPa in shear. The gears C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

Given data : AC = 250 mm ; BD = 400 mm ; D_C = 600 mm or R_C = 300 mm ; D_D = 200 mm or R_D = 100 mm ; AB = 2400 mm ; P = 20 kW = 20 × 10^3 W ; N = 120 r.p.m ; σ_t = 100 MPa = 100 N/mm^2 ; τ = 56 MPa = 56 N/mm^2 ; W_C = 950 N ; W_D = 350 N ; K_m = 1.5 ; K_t = 1.2



The shaft supported in bearings and carrying gears is shown in Fig.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590 \text{ N-m} = 1590 \times 10^3 \text{ N-mm}$$

Since the torque acting at gears *C* and *D* is same as that of the shaft, therefore the tangential force acting at gear *C*,

$$F_{tC} = \frac{T}{R_C} = \frac{1590 \times 10^3}{300} = 5300 \text{ N}$$

and total load acting downwards on the shaft at *C*

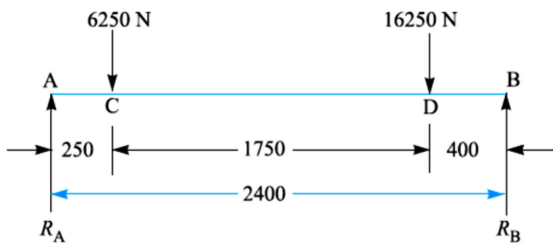
$$= F_{tC} + W_C = 5300 + 950 = 6250 \text{ N}$$

Similarly tangential force acting at gear *D*,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15\,900 \text{ N}$$

and total load acting downwards on the shaft at *D*

$$= F_{tD} + W_D = 15\,900 + 350 = 16\,250 \text{ N}$$



Now assuming the shaft as a simply supported beam as shown in Fig, the maximum bending moment may be obtained as discussed below :

Let R_A and R_B = Reactions at *A* and *B* respectively.

∴ $R_A + R_B$ = Total load acting downwards at *C* and *D*

$$= 6250 + 16\,250 = 22\,500 \text{ N}$$

Now taking moments about *A*,

$$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 10^3$$

∴ $R_B = 34\,062.5 \times 10^3 / 2400 = 14\,190$ N

and $R_A = 22\,500 - 14\,190 = 8310$ N

A little consideration will show that the maximum bending moment will be either at *C* or *D*. We know that bending moment at *C*,

$$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3 \text{ N-mm}$$

Bending moment at *D*,

$$M_D = R_B \times 400 = 14\,190 \times 400 = 5676 \times 10^3 \text{ N-mm}$$

∴ Maximum bending moment transmitted by the shaft,

$$M = M_D = 5676 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 5676 \times 10^3)^2 + (1.2 \times 1590 \times 10^3)^2}$$

$= 8725 \times 10^3$ N-mm

We also know that the equivalent twisting moment (T_e),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\, d^3$$

∴ $d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3$ or $d = 92.5$ mm

Again we know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}(K_m \times M + T_e)$$

We also know that the equivalent bending moment (M_e),

$$8620 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3 \quad \text{...(Taking } \sigma_b = \sigma_t\text{)}$$

∴ $d^3 = 8620 \times 10^3 / 9.82 = 878 \times 10^3$ or $d = 95.7$ mm

Taking the larger of the two values, we have

$d = 95.7$ say 100 mm.

Design of Shafts on the basis of Rigidity:-

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G \cdot \theta}{L} \quad \text{or} \quad \theta = \frac{T \cdot L}{J \cdot G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \quad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity*. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment.

The lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Problem(11):- *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Given:- $P = 4$ kW = 4000 W; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \text{ X } \frac{\pi}{180} = 0.0044$ rad ;

$L = 1$ m = 1000 mm ; $G = 84$ GPa = 84×10^9 N/m² = 84×10^3 N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that

$$\frac{T}{J} = \frac{G \times \theta}{L} \quad \text{or} \quad J = \frac{T \times l}{G \times \theta}$$

$$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

∴ $d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$$\tau = 47\,740 / 8420 = 5.67 \text{ N/mm}^2 = 5.67 \text{ MPa.}$$

Problem(12):- *Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.*

Given data: $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1 / 2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

W_H = Cross-sectional area × Length × Density

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad \ldots\ldots\ldots\ldots\ldots(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad \ldots\ldots\ldots\ldots\ldots(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation **(*i*)** by equation **(*ii*)**, we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \qquad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \qquad ....(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \qquad ....(iv)$$

Dividing equation **(*iii*)** by equation **(*iv*)**, we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\,(1 - k^4)}{d^3} = \frac{(d_o)^3\,(1 - k^4)}{(d_o)^3} = 1 - k^4 \qquad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \qquad ....(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \qquad ....(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \qquad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375$$

UNIT-IV (DMM-I) Shaft Couplings

Introduction: - Coupling is necessary to connect one shaft to another or to couple a drive shaft to a driven shaft. When the coupling can be arranged so as not to transmit torsional moment, it becomes a clutch. The essential difference between a clutch and a coupling is that a clutch can be disengaged or engaged at the will of the operator, where as a coupling is regarded as being fixed.

Purposes of a coupling(What are the various Purposes of a coupling?)

a). To transmit torque from one shaft to another.

b). To compensate for misalignment between the two shafts.

c). To provide smoother operations.

d). To introduce protection against overloads.

e). To alter the vibration characteristics of rotating units.

Requirements of a good shaft coupling (What are the requirements of a good shaft coupling?)

a). It should be easy to connect and disconnect,

b). It should provide perfect alignment of shafts,

c). It should avoid transmission of shock loads between shafts,

d). It should transmit full power between shafts without losses.

Classification of Shafts Couplings **(classify the coupling and explain them)**

Shaft couplings are divided into two main groups as follows:

1. ***Rigid coupling*****.** It is used to connect two shafts which are perfectly aligned (Collinear shafrs).

(*a*) Sleeve or muff coupling.

(*b*) Clamp or split-muff or compression coupling, and

(*c*) Flange coupling: - i). Un protected type flange coupling

ii) Protected type flange coupling

iii) Marine type flange coupling

2. ***Flexible coupling*****.** It is used to connect two shafts having both lateral and angular misalignment.

(*a*) Bushed pin type coupling,

(*b*) Oldham coupling, and

(*c*) Universal coupling.

Application of couplings:- (Write the applications of couplings)

Rigid couplings

1). It can transmit more power when compared to flexible coupling of the same size.

2). It used for in applications that operate at high temperatures.

3). Smaller sized rigid couplings are used for high torque capacity and stiffness,

4). It contain the zero backlash.

Flexible couplings

1).Flexible couplings are highly used in steel industries, petrochemical industries, off-road vehicles and heavy machinery etc.

2).It is also used in office machines, servomechanisms, instrumentation and light machinery etc.

Difference between rigid and flexible couplings***

S.No	Rigid couplings	Flexible couplings
1	Rigid couplings cannot accommodate any misalignment between the shafts.	Flexible couplings can accommodate a slight misalignment between the shafts.
2	It is cannot absorb shocks and vibrations.	It is absorb shocks and vibrations.
3	It is simple and inexpensive.	It is not simple and expensive.
4	**Types:-** **(*a*)** Sleeve or muff coupling. **(*b*)** Clamp or split-muff or compression coupling, and **(*c*)** Flange coupling: - i). Un protected type flange coupling ii) Protected type flange coupling iii) Marine type flange coupling	**Types:-** **(*a*)** Bushed pin type coupling, **(*b*)** Universal coupling, and **(*c*)** Oldham coupling.

1.Rigid couplings:- It is used to connect two shafts which are perfectly aligned (Collinear shafrs).

(a) Sleeve or Muff-coupling

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key as shown in Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

The usual proportions of a cast iron sleeve coupling are as follows:

Outer diameter of the sleeve, $D = 2d + 13$ mm

Length of the sleeve, $L = 3.5\ d$

Where d = diameter of the shaft.

Design procedure

1. *Design for sleeve*

The sleeve is designed by considering it as a hollow shaft.

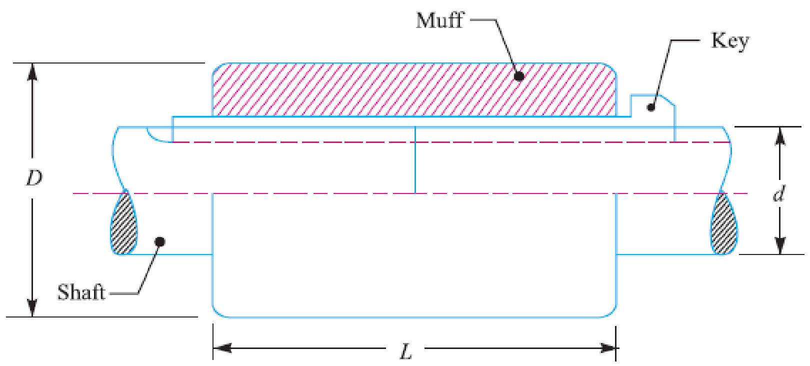
T = Torque to be transmitted by the coupling,

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \ (1 - k^4) \qquad \text{... } (\because \ k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.



2. *Design for key:-* The length of the coupling key is at least equal to the length of the sleeve (*i.e.* 3.5 *d*). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\ d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked.

Torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{... (Considering crushing of the key)}$$

Problem(1):-Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Given : $P = 40$ kW $= 40 \times 10^3$ W; $N = 350$ r.p.m.; $\tau_s = 40$ MPa $= 40$ N/mm^2;

$\sigma_{cs} = 80$ MPa $= 80$ N/mm^2; $\tau_c = 15$ MPa $= 15$ N/mm^2

The muff coupling is shown in Fig.

1. *Design for shaft*

Let *d* = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm}$$

2. *Design for sleeve*

Outer diameter of the muff,

$D = 2d + 13$ mm $= 2 \times 55 + 13 = 123$ say 125 mm.

And length of the muff,

$L = 3.5\, d = 3.5 \times 55 = 192.5$ say 195 mm.

Let us now check the induced shear stress in the muff. Let τc be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft.

Torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3\, \tau_c$$

$$\tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm2, therefore the design of muff is safe.

3. *Design for key*

Shaft diameter (d) = 55 mm,

Width of key, w = d/3, 55/3 =18 mm.

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w = 18$ mm.

We know that length of key in each shaft,

$l = L / 2 = 195 / 2 = 97.5$ mm.

Let us now check the induced shear and crushing stresses in the key.

Consider shearing of the key:- Torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\ \tau_s$$

$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8\ \text{N/mm}^2$$

Consider crushing of the key:- Torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3\ \sigma_{cs}$$

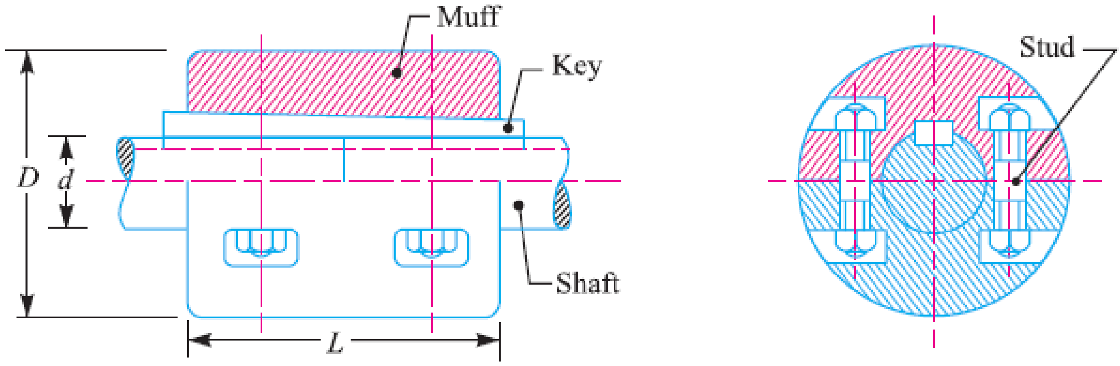
$$\sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6\ \text{N/mm}^2$$

Since the induced shear (22.8N/mm^2) and crushing stresses(45.6N/mm^2) are less than the permissible stresses(40MPa and 80Mpa), therefore the design of key is safe.

(b) Split muff coupling (Clamp or Compression Coupling):-

Muff or sleeve is made into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron. The shaft ends are made to about each other and a single key is fitted directly in the keyways of both the shafts. Both the halves are held together by means of mild steel studs or bolts and nuts.

This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling.



Design Procedure:-

Diameter of the muff or sleeve, $D = 2d + 13$ mm

Length of the muff or sleeve, $L = 3.5\ d$

Where d = Diameter of the shaft.

The power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft.

1. *Design of muff and key*

The muff and key are designed in the similar way as discussed in muff coupling.

2. *Design of clamping bolts*

Let T = Torque transmitted by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of friction between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

$\therefore$ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

$\therefore$ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2} \pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Note:- The value of μ may be taken as 0.3.

Problem(2):-*Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70*

MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.

Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. Design for shaft:-

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm}$$

2. Design for muff :-

Diameter of muff, $D = 2d + 13$ mm $= 2 \times 75 + 13 = 163$ say 165 mm.

Total length of the muff, L = 3.5 d = 3.5 × 75 = 262.5 mm.

3. Design for key :-

Width of key, w = d/3 = 75/3 = 25 mm.

Thickness of key, t = d/4 = 75/4 =18.75 mm.

Length of key = Total length of muff = 262.5 mm.

4. *Design for bolts:-*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu\,(d_b)^2\,\sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\,(d_b)^2\; 70 \times 6 \times 75 = 5830 (d_b)^2$$

$$(d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

Standard diameter of the bolt d_b = 27 mm (M 27).

(C) Flange Couplings

It consists of two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. One of the flange has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. This is used for heavy loads.The flange couplings are three types :

(a). Unprotected type flange coupling. (b). Protected type flange coupling.

(c). Marine type flange coupling.

(i). Unprotected type flange coupling.

d = diameter of the shaft or inner diameter of the hub, Outside diameter of hub, $D = 2\ d$

Length of hub, $L = 1.5\ d$

Pitch circle diameter of bolts, $D_1 = 3d$

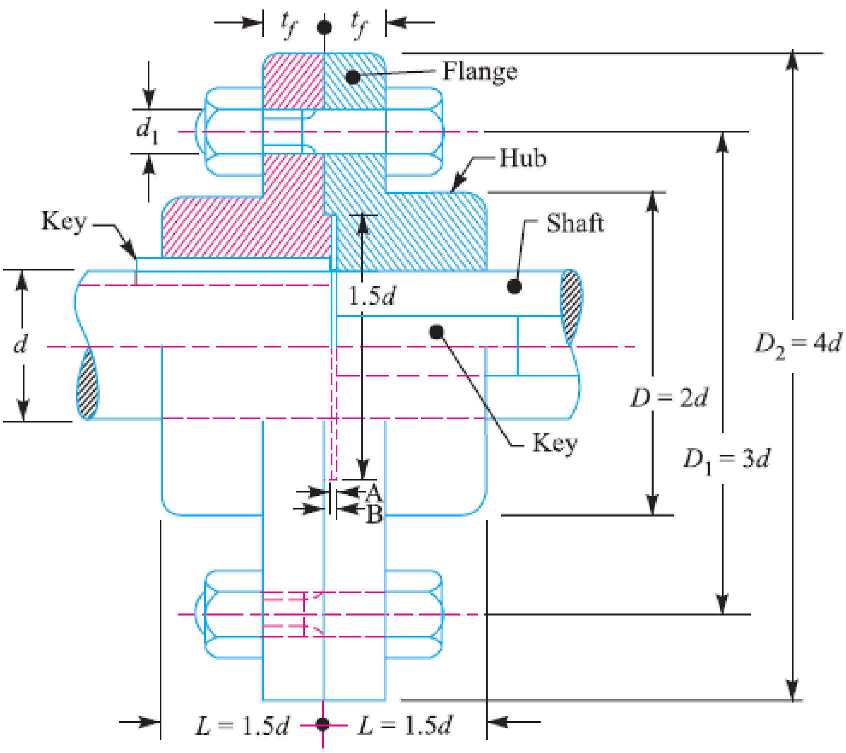
Outside diameter of flange, $D_2 = D_1 + (D_1 - D) = 2\ D_1 - D = 4\ d$

Thickness of flange, $t_f = 0.5\ d$

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto 180 mm



Unprotected type flange coupling

Design of Flange Coupling

Consider a flange coupling as shown in Fig.

Let d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

d_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively

τ_c = Allowable shear stress for the flange material *i.e.* cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

1. Design for hub

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

Where, D = 2d, L = 1.5d

Induced shearing stress in the hub may be checked from the above equation.

2. *Design for key*

The key is designed with usual proportions and then checked for shearing and crushing stresses. The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. *Design for flange*

The troque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub.

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. *Design for bolts*

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d.

$$\text{Load on each bolt} = \frac{\pi}{4} (d_1)^2 \tau_b$$

$\therefore$ Total load on all the bolts

$$= \frac{\pi}{4} (d_1)^2 \tau_b \times n$$

and torque transmitted, $$T = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f) \sigma_{cb}$$

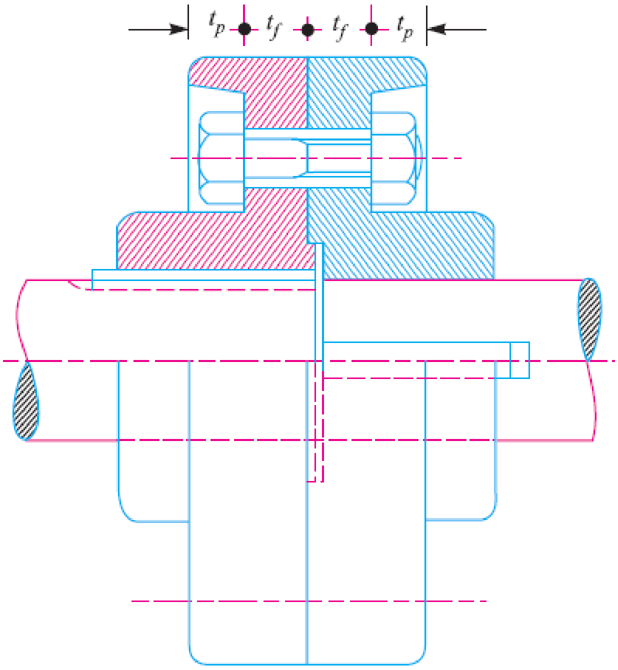
$\therefore$ Torque, $$T = (n \times d_1 \times t_f \times \sigma_{cb}) \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

(ii). Protected type flange coupling.

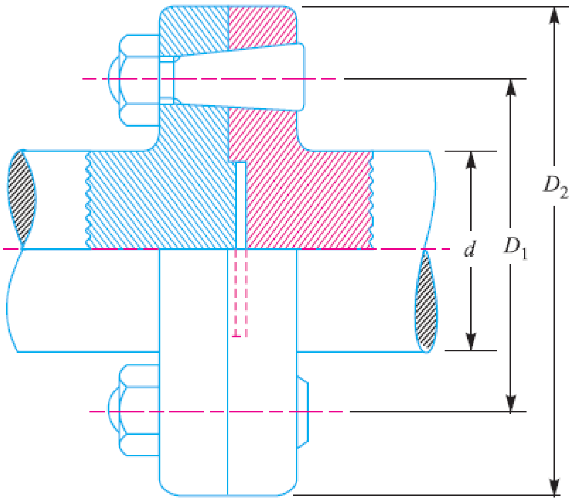
In a protected type flange coupling, as shown in Fig. The bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.

The thickness of the protective circumferential flange (t_p) is taken as 0.25 d. The other proportions of the coupling are same as for unprotected type flange coupling.



Protected type flange coupling

(iii). Marine type flange coupling. In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft.



Marine type flange coupling

The other proportions for the marine type flange coupling are taken as follows :

Thickness of flange = $d / 3$

Taper of bolt = 1 in 20 to 1 in 40

Pitch circle diameter of bolts, $D_1 = 1.6\, d$

Outside diameter of flange, $D_2 = 2.2\, d$

Problem(3):- Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :
Shear stress for shaft, bolt and key material = 40 MPa, Crushing stress for bolt and key = 80 MPa, Shear stress for cast iron = 8 MPa. Draw a neat sketch of the coupling.

Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 900$ r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k = 40$ MPa $= 40$ N/mm^2 ; $\sigma_{cb} = \sigma_{ck} = 80$ MPa $= 80$ N/mm^2 ; $\tau_c = 8$ MPa $= 8$ N/mm^2

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$T_{max} = 1.35 \times 159.13 = 215$ N-m $= 215 \times 103$ N-mm

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm}$$

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm}$$

and length of hub, $L = 1.5\, d = 1.5 \times 35 = 52.5$ mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\ 147\ \tau_c$$

$$\tau_c = 215 \times 10^3 / 63\ 147 = 3.4 \text{ N/mm}^2 = 3.4 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter, Width of key, $w = 12$ mm

and thickness of key, $t = w = 12$ mm

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L = 52.5$ mm

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\ \tau_k$$

$$\tau_k = 215 \times 10^3/11\,025 = 19.5 \text{ N/mm}^2 = 19.5 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

$$\sigma_{ck} = 215 \times 10^3/ 5512.5 = 39 \text{ N/mm}^2 = 39 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

$\therefore$ $t_f = 0.5\ d = 0.5 \times 35 = 17.5$ mm

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

Maximum torque transmitted (Tmax),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\,713\ \tau_c$$

$$\tau_c = 215 \times 10^3/134\,713 = 1.6 \text{ N/mm}^2 = 1.6 \text{ MPa}$$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is
safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts, n = 3

and pitch circle diameter of bolts, $D_1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted.

Maximum torque transmitted (Tmax),

$$215 \times 10^3 = \frac{\pi}{4} (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$$(d_1)^2 = 215 \times 10^3/4950 = 43.43 \text{ or } d_1 = 6.6 \text{ mm}$$

Assuming coarse threads, the nearest standard size of bolt is M 8.

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 35 = 140 \text{ mm}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 35 = 8.75 \text{ say } 10 \text{ mm}$$

Problem(4):- Design and draw a cast iron flange coupling for a mild steel shaft transmitting 90 kW at 250 r.p.m. The allowable shear stress in the shaft is 40 MPa and the angle of twist is not to exceed 1° in a length of 20 diameters. The allowable shear stress in the coupling bolts is 30 MPa.

Given : $P = 90$ kW $= 90 \times 10^3$ W ; $N = 250$ r.p.m. ; $\tau_s = 40$ MPa $= 40$ N/mm^2 ;

$\theta = 1° = \pi / 180 = 0.0175$ rad ; $\tau_b = 30$ MPa $= 30$ N/mm^2

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{90 \times 10^3 \times 60}{2\,\pi \times 250} = 3440 \text{ N-m} = 3440 \times 10^3 \text{ N-mm}$$

Considering strength of the shaft, we know that

$$\frac{T}{J} = \frac{\tau_s}{d/2}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{40}{d/2} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{80}{d} \quad \ldots \left(\because J = \frac{\pi}{32} \times d^4\right)$$

$$d^3 = 35 \times 10^6 / 80 = 0.438 \times 10^6 \quad \text{or} \quad d = 76 \text{ mm}$$

Considering rigidity of the shaft, we know that

$$\frac{T}{J} = \frac{C \times \theta}{l}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{84 \times 10^3 \times 0.0175}{20\,d} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{73.5}{d} \quad \ldots (\text{Taking } C = 84 \text{ kN/mm}^2)$$

$$d^3 = 35 \times 10^6 / 73.5 = 0.476 \times 10^6 \quad \text{or} \quad d = 78 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 78 \text{ say } 80 \text{ mm}$$

Design Procedure: Same the above problem

1. Design for hub, 2. Design for key, 3. Design for flange, 4. Design for bolts

Problem(5):-The shaft and the **flange of a marine engine** are to be designed for flange coupling, in which the flange is forged on the end of the shaft. The following particulars are to be considered in the design :

Power of the engine = 3 MW
Speed of the engine = 100 r.p.m.
Permissible shear stress in bolts and shaft = 60 MPa
Number of bolts used = 8
Pitch circle diameter of bolts = 1.6 × Diameter of shaft

Find : 1). diameter of shaft ; 2). diameter of bolts ; 3). thickness of flange ; and 4). diameter of flange. 5). Draw neat sketch of the coupling.

Given : $P = 3$ MW $= 3 \times 10^6$ W ; $N = 100$ r.p.m. ; $\tau_b = \tau_s = 60$ MPa $= 60$ N/mm^2 ; $n = 8$; $D_1 = 1.6\,d$

1. Diameter of shaft

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{3 \times 10^6 \times 60}{2\pi \times 100} = 286 \times 10^3 \text{ N-m} = 286 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$286 \times 10^6 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$d^3 = 286 \times 10^6 / 11.78 = 24.3 \times 10^6$$

$$d = 2.89 \times 10^2 = 289 \text{ say } 300 \text{ mm}$$

2. Diameter of bolts

Let d_1 = Nominal diameter of bolts.

The bolts are subjected to shear stress due to the torque transmitted.

Torque transmitted (T),

$$286 \times 10^6 = \frac{\pi}{4}(d_1)^2\, \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times (d_1)^2\, 60 \times 8 \times \frac{1.6 \times 300}{2}$$

$$= 90\,490\,(d_1)^2 \qquad \ldots (\because D_1 = 1.6\,d)$$

$$(d_1)^2 = 286 \times 10^6 / 90\,490 = 3160 \quad \text{or} \quad d_1 = 56.2 \text{ mm}$$

Assuming coarse threads, the standard diameter of the bolt is 60 mm (M 60). The taper on the bolt may be taken from 1 in 20 to 1 in 40.

3. Thickness of flange

The thickness of flange (t_f) is taken as d / 3.

∴ t_f = d / 3 = 300/3 = 100 mm.

Let us now check the induced shear stress in the flange by considering the flange at the junction of the shaft in shear.

Torque transmitted (T),

$$286 \times 10^6 = \frac{\pi d^2}{2} \times \tau_s \times t_f = \frac{\pi(300)^2}{2} \times \tau_s \times 100 = 14.14 \times 10^6 \tau_s$$

$$\tau_s = 286 \times 10^6 / 14.14 \times 10^6 = 20.2 \text{ N/mm}^2 = 20.2 \text{ MPa}$$

Since the induced shear stress in the flange is less than the permissible shear stress of 60 MPa, therefore the thickness of flange (t_f = 100 mm) is safe.

4. Diameter of flange

The diameter of flange (D_2) is taken as 2.2 d.

∴ $D_2 = 2.2\ d = 2.2 \times 300 = 660$ mm.

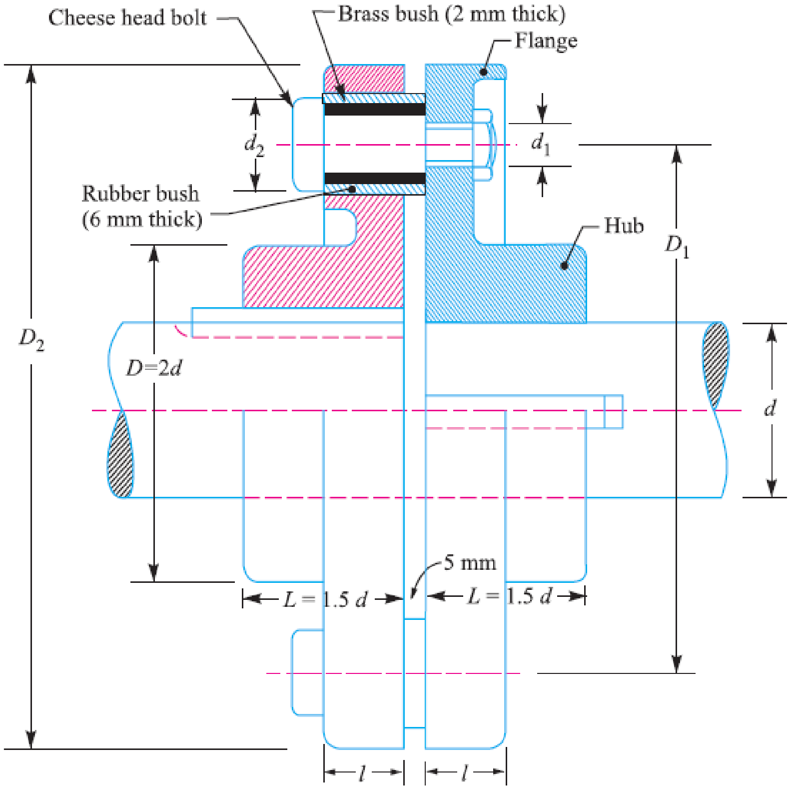
(2) Flexible Couplings:- It is used to connect two shafts having both lateral and angular misalignment.

(*a*) Bushed pin type coupling,

(*b*) Oldham coupling and

(*c*) Universal coupling.

(a) Bushed-pin Flexible Coupling



Bushed-pin Flexible Coupling

A bushed-pin flexible coupling, as shown in Fig. is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling.

Let l = Length of bush in the flange,

d_2 = Diameter of bush,

p_b = Bearing pressure on the bush or pin,
n = Number of pins, and
D_1 = Diameter of pitch circle of the pins.

We know that bearing load acting on each pin,

$$W = p_b \times d_2 \times l$$

∴ Total bearing load on the bush or pins

$$= W \times n = p_b \times d_2 \times l \times n$$

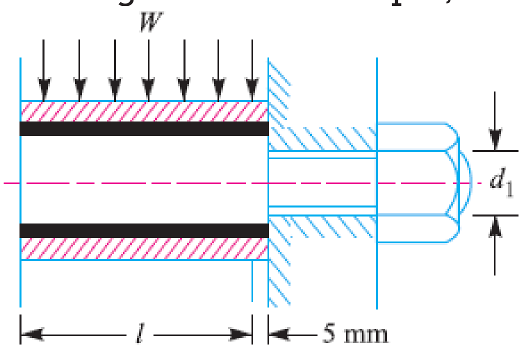
And the torque transmitted by the coupling,

$$T = W \times n\left(\frac{D_1}{2}\right) = p_b \times d_2 \times l \times n\left(\frac{D_1}{2}\right)$$

Direct shear stress due to pure torsion in the coupling halves

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$

Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin as shown in Fig. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,



$$M = W\left(\frac{l}{2} + 5 \text{ mm}\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5 \text{ mm}\right)}{\frac{\pi}{32}(d_1)^3}$$

Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress by the following relations:

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.

Note: After designing the pins and rubber bush, the hub, key and flange may be designed in the similar way as discussed for flange coupling.

Problem(6):-Design a **bushed-pin type of flexible coupling** to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque. The material properties are as follows :

(a) The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.

(b) The allowable shear stress for cast iron is 15 MPa.

(c) The allowable bearing pressure for rubber bush is 0.8 N/mm^2.

(d) The material of the pin is same as that of shaft and key.

Draw neat sketch of the coupling.

Given : $P = 32$ kW $= 32 \times 10^3$ W ; $N = 960$ r.p.m. ; $T_{max} = 1.2\, T_{mean}$; $\tau_s = \tau_k = 40$ MPa $= 40$ N/mm^2 ; $\sigma_{cs} = \sigma_{ck} = 80$ MPa $= 80$ N/mm^2 ; $\tau_c = 15$ MPa $= 15$ N/mm^2 ; $p_b = 0.8$ N/mm^2

1. <u>Design for pins and rubber bush</u>

Find the diameter of the shaft (d). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

The rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins (n) as 6.

$$\text{Diameter of pins, } d_1 = \frac{0.5\, d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

Bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 24 mm.
The length of the pin of least diameter i.e. d_1 = 24 mm
Assume the brass bush of thickness 2 mm. and thickness of rubber bush as 6 mm.

∴ Overall diameter of rubber bush,

$$d_2 = 24 + 2 \times 2 + 2 \times 6 = 40 \text{ mm.}$$

Diameter of the pitch circle of the pins

$$D_1 = 2\,d + d_2 + 2 \times 6$$
$$= 2 \times 40 + 40 + 12 = 132 \text{ mm}$$

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\,l \text{ N}$$

and the maximum torque transmitted by the coupling (T_{max}),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} = 32\,l \times 6 \times \frac{132}{2} = 12\,672\,l$$

$$l = 382 \times 10^3 / 12\,672 = 30.1 \text{ say } 32 \text{ mm}$$

$$W = 32\,l = 32 \times 32 = 1024 \text{ N}$$

∴ Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

We know that bending stress,

$$\sigma = \frac{M}{Z}$$

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\,504 \text{ N-mm}$$

and section modulus, $Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{21\,504}{785.5} = 27.4 \text{ N/mm}^2$$

∴ Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\,(3.26)^2}\right]$$
$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

and maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\,(3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. <u>Design for hub</u>

Outer diameter of the hub, D = 2 d = 2 × 40 = 80 mm

and length of hub, L = 1.5 d = 1.5 × 40 = 60 mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3\, \tau_c$$

$$\tau_c = 382 \times 10^3 \,/\, 94.26 \times 10^3 = 4.05 \text{ N/mm}^2 = 4.05 \text{ MPa}$$

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. <u>Design for key</u>

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\,\tau_k$), therefore a square key may be used. From Table for a shaft of 40 mm diameter,

Width of key, w = 14 mm

and thickness of key, $t = w = 14$ mm

The length of key (L) is taken equal to the length of hub, *i.e.*

$$L = 1.5\, d = 1.5 \times 40 = 60 \text{ mm}$$

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\,800\, \tau_k$$

$$\tau_k = 382 \times 10^3/16\,800 = 22.74 \text{ N/mm}^2 = 22.74 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\, \sigma_{ck}$$

$$\sigma_{ck} = 382 \times 10^3/8400 = 45.48 \text{ N/mm}^2 = 45.48 \text{ MPa}$$

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. <u>Design for flange</u>

The thickness of flange (t_f) is taken as 0.5 d.

$$\therefore \quad t_f = 0.5\, d = 0.5 \times 40 = 20 \text{ mm}$$

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

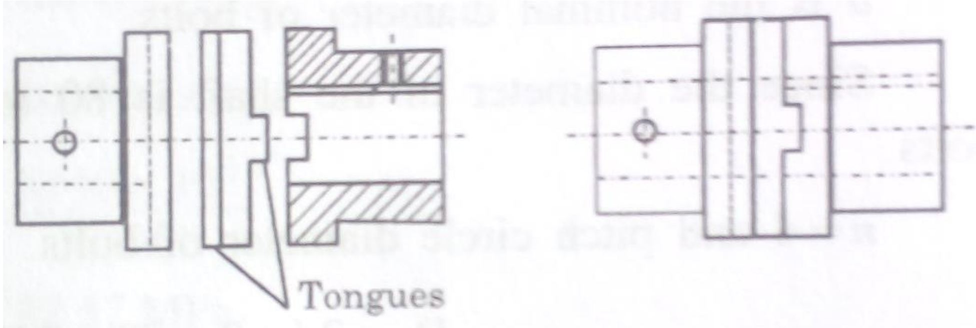
$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3\, \tau_c$$

$$\tau_c = 382 \times 10^3 / 201 \times 10^3 = 1.9 \text{ N/mm}^2 = 1.9 \text{ MPa}$$

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

(b)Oldham Coupling:-

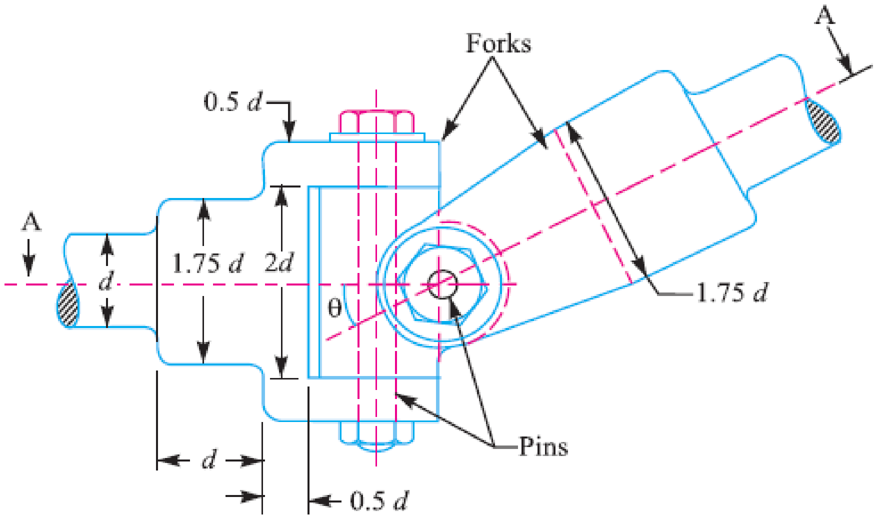
This coupling is used to connect shafts that have lateral misalignment. This coupling consists of two slotted hubs and a central floating member. The floating member has two tongues, one on each face, at right angles to each other. These tongues slide in the slots of the hubs. The Oldham coupling as shown in the figure.



Oldham coupling

(c) Universal (or Hooke's) Coupling:-

An universal coupling is used to connect two shafts whose axes are inclined to each other at small angle. Main application of universal joint is found in transmission from gear box to differential or back axle of automobiles. It is also used in transmission of multiple spindle drilling machines. It is used as a knee Joint in milling machines. Universal coupling as shown in the figure.



Problem(7):- A torque of 3000 Nm is to be transmitted between two shafts by using a universal coupling. Shaft is transmitting torque only. Find the diameter of the shaft and pin. Take shear stress for shaft is 50 MPa and shear stress for pin is 30 MPa. If angle of inclination of shafts is 15^0 and in plane angle driving shaft with shaft fork is 20^0. Find the speed of output shaft and speed of input shaft is 1200 rpm.

Solution: Given Data:

$T = 3000$ Nm, $[\tau_s] = 50$ MPa $= 50$ N/mm^2, $\tau_{pin} = 30$ MPa $= 30$ N/mm^2

$\theta = 20°$, $\alpha = 15°$, $N_1 = 1200$ rpm

1. Diameter of shaft (d)

We know that

$$\text{Torque } (T) = \frac{\pi}{16}[\tau_s]\, d^3$$

$$3000 \times 10^3 = \frac{\pi}{16} \times 50 \times d^3$$

$$d^3 = \frac{3000 \times 10^3 \times 16}{\pi \times 50} = 305578;\ d = 67.35 \text{ mm}$$

From R20, standard diameter of shaft = 70 mm.

2. Diameter of pin (d_p)

$$\text{Again, Torque } (T) = 2 \times \frac{\pi}{16} \times [\tau_p] \times d_p^2 \times d$$

$$3000 \times 10^3 = 2 \times \frac{\pi}{16} \times 30 \times d_p^2 \times 70$$

$$\therefore \; d_p^2 = \frac{3000 \times 10^3 \times 16}{2 \times \pi \times 30 \times 70} = 3637.8$$

$$d_p = 60.314 \text{ mm}$$

R20 series standard diameter $d_p = 65$ mm.

3. Speed of output shaft (N_2)

We know that

$$\frac{N_1}{N_2} = \frac{1 - \cos^2\theta \sin^2\alpha}{\cos\alpha} \quad \text{(or)} \quad \frac{1200}{N_2} = \frac{1 - \cos^2 20 \sin^2 15}{\cos 15}$$

Speed of output shaft $N_2 = 1502.4$ rpm.

D.SrinivasuluReddy, M.Tech, M.I.S.T.E, (Ph.D), Associate Professor, Mechanical Engineering Dept., K.H.I.T, Guntur.